



# Studies of fast protons produced in the reactions of $^{16}\text{O}$ Nuclei incident on emulsion nuclei at 3.7A GeV

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## ARTICLE INFO

### Keywords:

3.7 GeV  $^{16}\text{O}$ -Em interactions  
Fast protons  
Multiplicity distributions  
Impact parameter  
Various moments  
Centrality

## ABSTRACT

We report the experimental data of the fast target protons produced in the reaction of  $^{16}\text{O}$ -Em at 3.7A GeV. The emission mechanism of the fast target protons (grey particles with kinetic energy in the range 26 – 400 MeV) is studied in terms of multiplicity characteristics. Our experimental data is compared with that of  $^{12}\text{C}$ ,  $^{22}\text{Ne}$  and  $^{28}\text{Si}$  projectile interactions at nearly the same incident energy. The results reveal that the multiplicity distribution is independent of both the projectile energy and its size. The interaction events were divided into two categories. The first depends on the number of nucleons remaining as spectators  $Q$  (stripped nucleons) from the projectile, while in the second category the interactions are classified according to the target size ( $N_{\text{h}}$ - parameter). The higher moments of the multiplicity distributions were calculated statistically for various groups of events. The experimental data show that the selectivity of  $N_{\text{h}}$  particle multiplicity as an indicator for nuclear interaction impact parameter is better as compared to the choice of the charged projectile spectator nucleons  $Q$  in each event.

## 1. Introduction

One of the important aspects of high energy hadron-nucleus (h-A) and nucleus-nucleus (A-A) reactions is the target fragmentation region, in which particles with  $Z \leq 2$  are emitted in  $4\pi$  space. To gain a complete understanding of the reaction mechanisms in (A-A) collisions, the target fragmentation region is usually studied exclusively. The outcome of an A-A event taking place in a photographic nuclear emulsion terminology, which provides a  $4\pi$  angular coverage, are termed as shower, grey, or black tracks [1, 2]. The EMOU1 – Collaboration [3, 4] investigated the grey tracks, which are mostly fast target protons with energy in the range  $26 < E < 400$  MeV, in the framework of two Monte Carlo simulation codes, namely the multichain model by Ranft [3] and Lund model Fritiof [4]. These codes completely disregard the target cascade, which is an important parameter for emission of the slow target fragmented particles (black tracks). Hegab et al. [5] demonstrated that the grey multiplicity distributions in (AA) collisions can be fitted with a geometrical model in which the input information is the multiplicity of grey particles produced in proton-nucleus (PA) interaction. As we know, the most significant observation to explain the particle production mechanism is the observation of the fast proton multiplicity [5]. Also, the multiplicity distribution (MD) carries information about the correlation of the emitted particles. The MD shape has been well described with the modified statistical model based on the Maxwell Boltzmann distribution [6–7], where the moments refer to the collection of mathematical functions associated with the probability distribution.

The present work is a continuation program for studying the characteristics and mechanisms of the grey particles production of

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**Table 1**  
Chemical composition of the used emulsion type.

Element	$^1\text{H}$	$^{12}\text{C}$	$^{14}\text{N}$	$^{16}\text{O}$	$^{80}\text{Br}$	$^{108}\text{Ag}$
(atoms/cm $^3 \times 10^{22}$ )	3.15	1.410	0.395	0.956	1.028	1.028

**Table 2**  
The percentage of events at different target size for different projectiles ( $^{12}\text{C}$ ,  $^{16}\text{O}$ ,  $^{22}\text{Ne}$  and  $^{28}\text{Si}$ ).

Projectile	$^{12}\text{C}$	$^{16}\text{O}$	$^{22}\text{Ne}$	$^{28}\text{Si}$
Incident energy (A GeV)	3.7	3.7	3.3	3.7
Total number of inelastic interaction	819	1725	3812	1029
Percentage of interaction with (CNO nuclei)	35.60	45.62	37.93	38.37
Percentage of interaction with AgBr nuclei.	64.40	54.38	62.07	61.63
$N_h \leq 7$	-	55.15	-	-
$N_h \geq 8$	-	44.85	-	-
$N_h \geq 28$	12.43	13.5	-	-

different projectiles interacting with emulsion nuclei. The multiplicities of the grey particles emitted in  $^{16}\text{O}$ -Em interaction at 3.7A GeV are statistically investigated in terms of different moments, where the statistical moments reflect the dynamics of the interaction [8–10]. The measured data of the grey particle multiplicity distribution has been studied as a function of the centrality and with different target sizes. We also investigate the various moments (mean, variance, kurtosis, and skewness) of the multiplicity distribution (MD) for different groups of the grey particles.

The present paper is arranged as follows: in Section 2, we demonstrate the experimental procedures and the selection criteria that have been applied. Section 3 presents the experimental data with the results obtained from the MD groups and the discussion. Section 4 contains the final conclusions.

## 2. Experimental details

A stack of NIKFI-BR-2 emulsion pellicles with dimensions of 10cm  $\times$  20 cm  $\times$  60 $\mu\text{m}$  was exposed to a 3.7A GeV  $^{16}\text{O}$  beam at the Dubna synchrophastron – JINR (Russia). The stack has a sensitivity of about 30 grains per 100  $\mu\text{m}$  for singly charged minimum ionizing particles. Each of the stack pellicles were scanned under magnification 1500 x by along- the-track method fast in the forward direction and slowly in the backward direction using an 850056 STEINDORFF microscope. Each beam of the primary incident track was carefully observed until it escaped or interacted with the pellicle up to a potential path length of 7 cm from the beam entrance in the pellicle.

Other details about the experimental procedures and general characteristics have previously been published in ref. [11 , 12, 13, 14]. The chemical composition of the emulsion is listed in Table (1).

In all inelastic events, the emitted charged particles were classified according to the following conventional emulsion terminology:

- i Shower particles: are singly charged relativistic particles with relative ionization ( $I/I_0 \leq 1.4$ ), where  $I_0$  is the plateau ionization for singly charged minimum ionizing particles. These are mostly charged pions with speed  $\beta = V/C \geq 0.7$ . Their multiplicity is denoted by  $n_s$ .
- ii Grey particles: are particles having a range  $R > 3$  mm in emulsion and relative ionization ( $1.4 < I/I_0 < 4.5$ ). These particles are mainly protons with kinetic energy in the range 26–400 MeV. The multiplicity of those tracks is denoted by  $N_g$ .
- iii Black particles: are particles having relative ionization ( $I/I_0 \geq 4.5$ ) and having a range in emulsion  $R \leq 3$  mm. They are mainly the evaporation products of the remnant of the target nucleus, which are characterized by energy  $< 26$  MeV. Their multiplicity is denoted by  $N_b$ .

In each event, the grey and black particles are called heavily ionizing particles. They are mainly target fragments, and their multiplicity is denoted by  $N_h (=N_g + N_b)$ .

In each event, the charges of individual projectile fragments with  $Z \geq 2$  (PFs) were determined using a combined technique of grain gap and  $\delta$ -ray densities [15]. More details of the charge determination of all emitted PFs have previously been published in ref. [15].

In each event, the total charge of the projectile spectators  $= \sum_{i=1}^8 N_i Z_i$ , was calculated as well, where  $N_i$  is the number of emitted fragments having a charge  $Z_i$  ( $N_i$  takes a value from 1 to 8).

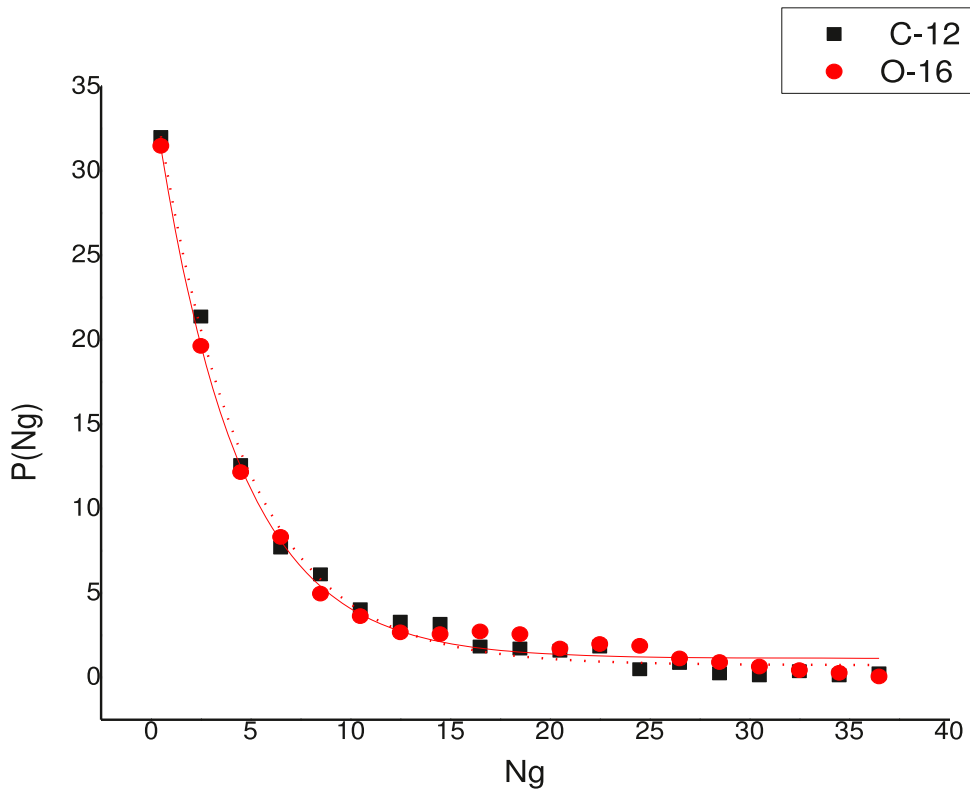
For the determination of the degree of centrality in each event, i.e., it is necessary to know the impact parameter between the centers of the interacting nuclei. This parameter determines the number of participant nucleons from both target and projectile in the interaction. The impact parameter cannot be measured experimentally. As a result, the sum of the number of stripped charged nucleons  $Q$  (number of projectile nucleon spectators) can be used as a selection criterion for the degree of collision centrality in each event [14, 15].  $Q$  in our projectile experiment ranges from zero to eight. Events with  $Q = 0$  are considered as pure central collisions, and the degree of centrality decreases as  $Q$  increases.

**Table 3**The various moments of MDs of the grey particles ( $M = \langle N_g \rangle$ ),  $\sigma^2$ , S and K for different projectiles at Dubna energies.

Projectile	$M = \langle N_g \rangle$	$\sigma^2$	S	K	(K-3)	$\omega$
$^{16}\text{O}$	$6.42 \pm 0.18$	$7.88 \pm 0.05$	$1.62 \pm 0.06$	$1.40 \pm 0.19$	- 1.6	1.23
$^{12}\text{C}$	$5.68 \pm 0.20$	$6.75 \pm 0.06$	$1.80 \pm 0.09$	$3.40 \pm 0.17$	0.40	1.19
$^{22}\text{Ne}$	$6.94 \pm 0.11$	$8.52 \pm 0.03$	$1.72 \pm 0.06$	$2.25 \pm 0.08$	- 0.75	1.23
$^{28}\text{Si}$	$6.94 \pm 0.20$	$8.47 \pm 0.06$	$1.71 \pm 0.02$	$2.69 \pm 0.13$	-0.31	1.22

**Table 4**The values of various moments ( $\langle N_g \rangle$ , S and K) and the variance.

$N_h$ - values	$\langle N_g \rangle$	$\sigma^2$	S	K	K-3	$\omega$
$N_h < 8$	$2.23 \pm 0.08$	$1.53 \pm 0.03$	$-6.19 \pm 0.09$	$-450.60 \pm 0.18$	- 453.60	0.65
$N_h \geq 8$	$12.55 \pm 0.43$	$9.66 \pm 0.08$	$-0.52 \pm 0.08$	$1.35 \pm 0.17$	- 1.65	0.59
$N_h (8 - 15)$	$5.32 \pm 0.30$	$2.22 \pm 0.06$	$1.29 \pm 0.14$	$40.10 \pm 0.28$	37.10	0.42
$N_h (16 - 27)$	$11.65 \pm 0.68$	$3.87 \pm 0.08$	$0.44 \pm 0.14$	$0.29 \pm 0.12$	- 2.71	0.33
$N_h \geq 28$	$23.36 \pm 1.53$	$5.81 \pm 0.11$	$0.76 \pm 0.16$	$0.98 \pm 0.32$	- 2.02	0.25

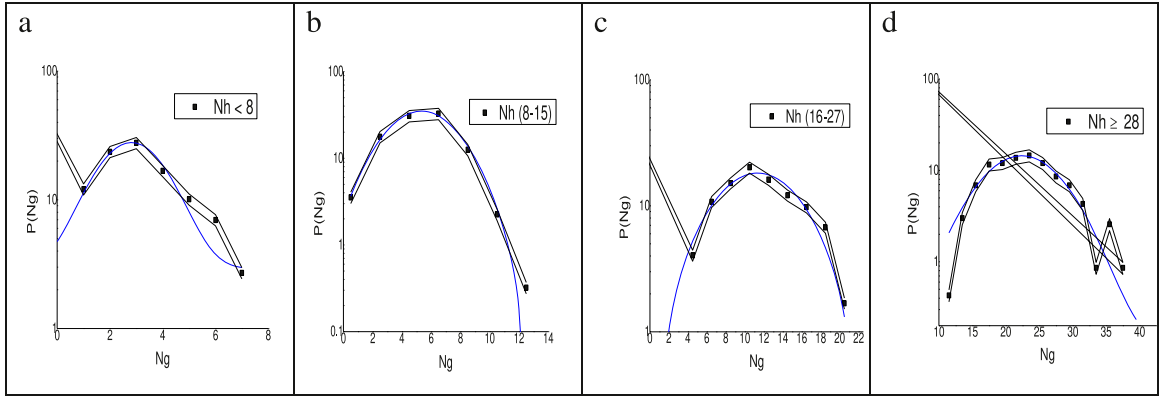
**Fig. 1.** Multiplicity distributions of the g-particles produced in the collisions of  $\text{O}^{16}\text{-Em}$  and  $\text{C}^{12}\text{-Em}$  at same energy 3.7 A GeV together with the fitting curves.

### 3. Results and discussion

#### 3.1. Target size dependence

The incident  $^{16}\text{O}$  ions are considered to interact with each one of the following groups of composite emulsion target: the free hydrogen H, CNO (the light group) or Ag Br (the heavy group). The effective masses of these groups are, 1, 14 and 94 respectively. There are various techniques for target nuclei discrimination. [16–21]

Table (2) shows the probabilities of observing interactions of 3.7A GeV  $^{16}\text{O}$  with different groups having  $N_h \leq 7$ ,  $N_h \geq 8$  and  $N_h \geq 28$ . Similar data for the interactions of  $^{12}\text{C}$ ,  $^{22}\text{Ne}$  and  $^{28}\text{Si}$  projectiles [22] with emulsion nuclei at Dubna energies are also listed for comparison. The events having  $N_h \leq 7$ , are considered to be due to interactions with the light group of nuclei in the emulsion (CNO) in



**Fig. 2.** Multiplicity distributions of the g-particles produced in the interactions of 3.7 A GeV  $^{16}\text{O-Em}$  at different target size  $N_h$ . The experimental data are fitted with the Gaussian distribution function.

addition to those due to peripheral interactions of (AgBr) nuclei. The events with  $N_h \geq 28$  in Table (2) are considered to be due to the complete destruction of AgBr group [23] in emulsion. This sample of interactions is characterized by central collisions.

The average multiplicity of the distribution of grey particles emitted in the interactions of  $^{16}\text{O-Em}$  is presented in Table (3). The dependence of  $\langle N_g \rangle$  on the projectile mass number, is investigated by comparing our previous experimental data with those obtained for the interactions of  $^{12}\text{C}$ ,  $^{22}\text{Ne}$ , and  $^{28}\text{Si}$  projectiles [22, 23] at nearly the same incident energy. The analysis of the higher moments of the MDs [24–30] reveals information about the collision mechanism and their departure from Poisson type distribution. Table (3) also includes the values of variance  $\sigma^2$ , skewness  $S$ , and kurtosis  $K$ . The excess kurtosis ( $=K-3$ ) and the scaled variance [ $\omega (= \sigma^2/M)$ ] are calculated and listed in the last two columns of Table (3).

Table (4) shows the data for the moments ( $M$ ,  $S$ , and  $K$ ) and the variances  $\sigma^2$  of MDs of the emitted grey particles in the  $^{16}\text{O-Em}$  interactions for different target sizes ( $N_h$  – values). The last two columns display the calculated excess Kurtosis ( $K-3$ ) and the scaled variance  $\omega (= \frac{\sigma^2}{M})$ . The MD of grey particles  $P(N_g)$  for  $^{16}\text{O-Em}$  interactions is shown in Fig. 1 in comparison to the data for  $^{12}\text{C-Em}$  interactions at the same incident energy (3.7A GeV). The experimental data has been best fitted for  $^{16}\text{O-Em}$  according to Eq. (1),

$$P(N_g) = ae^{(-N_g/b)} \quad (1)$$

where  $a = 37.15$  and  $b = 4.09$ .

In the case of  $^{12}\text{C}$  interactions with emulsion, the values of the fitting parameter  $a$  and  $b$  are  $a = 35.08$  and  $b = 4.40$ .

Figs. 2(a-d) represent the MDs of grey particles emitted in  $^{16}\text{O-Em}$  interactions for different target sizes  $N_h < 8$ ,  $N_h (8-15)$ ,  $N_h (16-27)$  and  $N_h \geq 28$  respectively.

From the analysis of the above data we notice that:

The Shapes of all  $N_h$  groups are systematic Gaussians. This demonstrates that  $N_h$  is deeply related to the impact parameter and can also be taken as an experimental estimate for this parameter.

According to Table (3), the values of the average multiplicities of the emitted grey particles  $M (= \langle N_g \rangle)$  and the calculated values of the different moments, variance  $\sigma^2$ , skewness  $S$ , kurtosis and the scaled variance  $\omega$  appear to be within the experimental errors independent of the projectile mass number. In addition, the skewness values are positive and greater than one, implying that the MD has highly skewed shape.

The values of the excess kurtosis ( $K-3$ ) are negative, i.e., the MD curves are of one platykurtic shape.

The values of the scaled variance  $\omega$  are nearly constant ( $\approx 1.2$ ), independent of the projectile size at the same incident energy. As a result, the MD curves of the four projectiles are of the same shape.

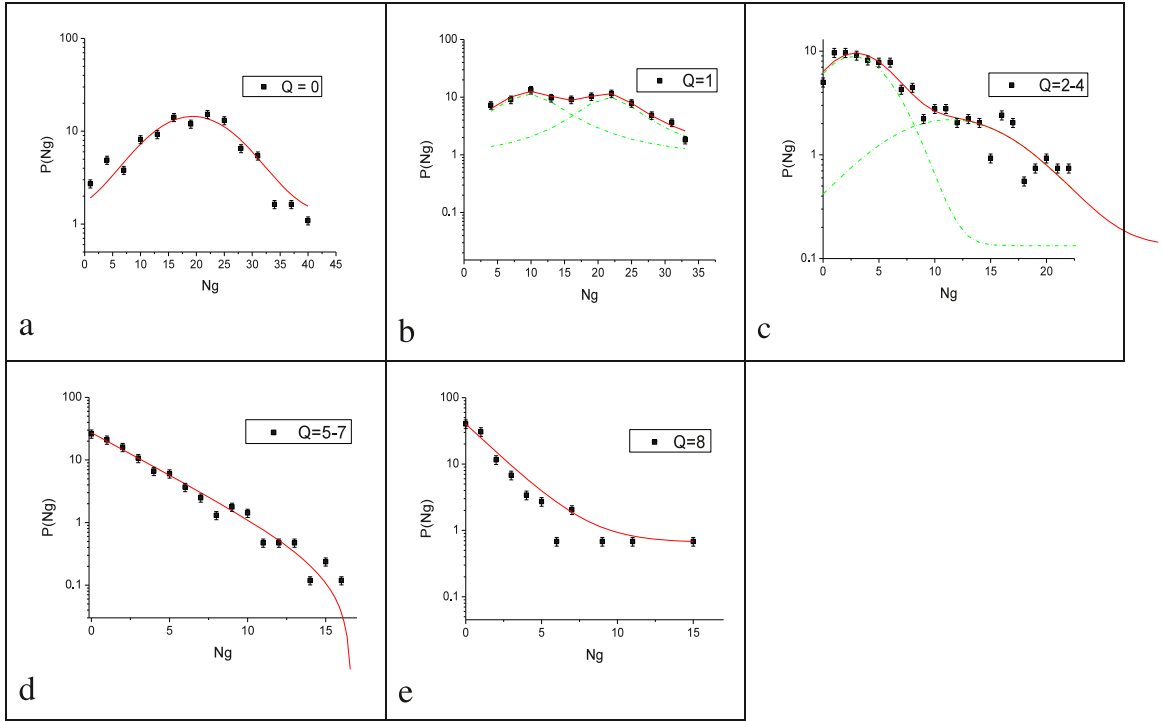
The calculated multiplicity moments ( $M$ ,  $S$ , and  $K$ ) for different target sizes  $N_h$  in Table (4) have different values, i.e., the characteristic shapes of the MD of the grey particles change from one to the other. The events with  $N_h < 8$ , corresponding to the interactions with the light group CNO nuclei as well as peripheral interactions with AgBr nuclei, are negative and highly skewed ( $S < -1$ ), indicating that the MD curve is left skewed. Also, the kurtosis,  $K$ , has a high negative value where the MD has the possibility of a platykurtic shape.

The distribution of the emitted grey particles in the group of events having  $N_h \geq 28$ , which is considered as pure central events with AgBr group in the emulsion, is nearly symmetric and takes on platykurtic distribution shape. Therefore, the skew  $S$  takes values between  $+1/2$  and  $\pm 1$ , and the  $K$ -value is nearly equal to  $+1$ . This leads us to conclude that the emitted grey particle distribution is flatter than the Gaussian curve.

The events with  $N_h (16-27)$ , which represent the nearly central and quasi-central interactions with the AgBr group, have a skew  $S$  value nearly equal to  $+1/2$  while the  $K$  value is less than 3.

The values of the scaled variance  $\omega$  decrease as the  $N_h$  values increase, i.e., with increasing target size.

Figs. 2 (a-d) shows that the data are well fitted with a Gaussian distribution. In Fig. 2a, the MD of grey particles is left-skewed (skew less than -1) and has a negative value of kurtosis. Also, its central peak gets wider and smaller. Furthermore, the distribution has a short tail.



**Fig. 3.** Multiplicity distributions of the g-particles produced in the interactions of 3.7 A GeV  $^{16}\text{O-Em}$  at different centralities Q, The experimental data are fitted with the a Gaussian distribution function.

**Table 5**

The various of various moments ( $\langle N_g \rangle$ , S and K) and the variance  $\sigma^2$  of the multiplicity distributions of the emitted g-particles at different Q-values.

Q	$\langle N_g \rangle$	$\sigma^2$	S	K	K-3	$\omega$
0	$17.87 \pm 1.32$	$8.50 \pm 0.15$	$-0.11 \pm 0.18$	$-0.17 \pm 0.36$	3.17	0.43
1	$14.80 \pm 1.15$	$9.02 \pm 0.17$	$0.35 \pm 0.19$	$-0.46 \pm 0.38$	-3.46	0.61
2–4	$6.86 \pm 0.30$	$5.81 \pm 0.07$	$1.37 \pm 0.11$	$2.63 \pm 0.21$	0.37	0.85
5–7	$2.66 \pm 0.09$	$3.27 \pm 0.04$	$1.54 \pm 0.08$	$-2.321 \pm 0.17$	-26.21	1.23
8	$1.37 \pm 0.11$	$2.26 \pm 0.09$	$3.83 \pm 0.20$	$21.81 \pm 0.20$	18.81	1.65

**Figs. 3b** and **3c** show that the distributions of grey particles in the groups of events with  $N_h$  (8-15 and 16–27) have a tendency to skew (S is between 0 and 1) and excess kurtosis  $> 0$ . These are nearly symmetric and platykurtic distributions.

### 3.2. Dependence on the impact parameter

We study here the dependence of the MD of the grey particles on the degree of centrality Q (projectile spectator charge), which may be related to the impact parameter. **Figs. 3(a-e)** show the MDs of grey particles  $P(N_g)$  emitted in 3.7A GeV  $^{16}\text{O-Em}$  interactions at different values of Q [Q = 0, Q = 1, Q (2-4), Q (5-7), and Q = 8], but do not provide a systematic result; only Q = 0 has a Gaussian shape. AS a result, the parameter Q cannot be taken as an experimental measure for the impact parameter between the interacting nuclei. This may be due to the difference between the probability of interaction of neutrons and that of protons. The number of noninteracting projectile protons, which is represented by Q, is not exactly equal to the number of noninteracting projectile neutrons in each event. These multiplicity distribution groups are further investigated statistically in terms of there higher moments which are listed in **Table (5)**.

According to **Fig. 3a**, the grey particles MD for events corresponding to Q = 0 (central collisions) is nearly of symmetrical shape (S is between  $-1/2$  and  $+1/2$ ) and  $K > 3$  (i.e., Platykurtic form).

The MDs of the grey particles from events having Q > 0 in **Figs. 3b** and **3c** show bimodal distributions with two individual peaks. This may be attributed to the emission of grey particles from two different sources (quasi- central collisions with the CNO group of nuclei and peripheral events with the AgBr group of nuclei in the emulsion). The data of these distributions are moderately skewed (S > 0) and platykurtic (K < 3).

**Figs. 3d** and **3e** show groups of events with Q (5–7) and Q = 8 that represent peripheral collisions with the emulsion nuclei and are exponentially decaying for both groups of events. The MDs are significantly skewed (S > + 1/2).

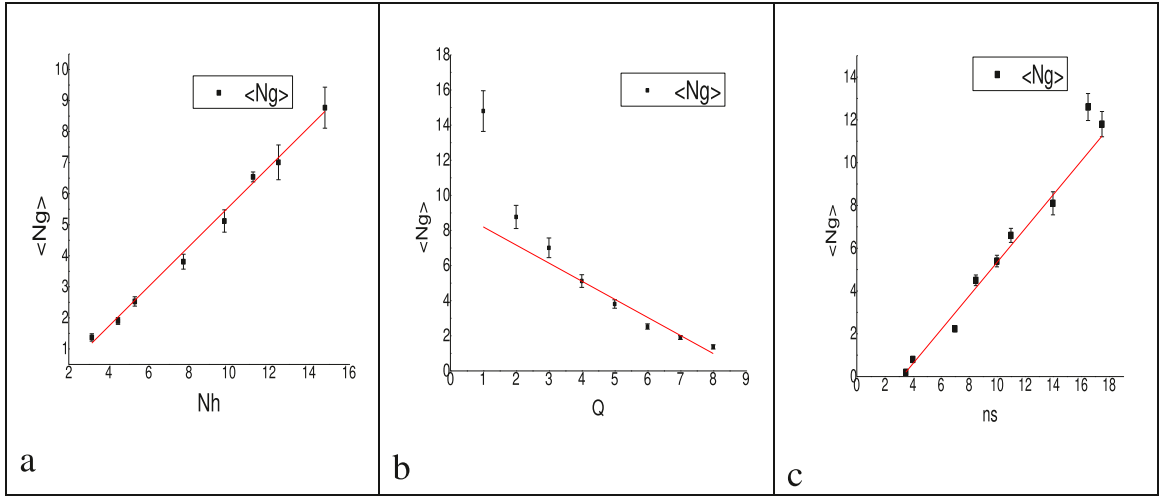


Fig. 4. The correlations between  $\langle N_g \rangle$  as function of  $N_h$ ,  $Q$  and  $n_s$  multiplicities.

**Table 6**  
The straight line fitting parameters of Fig. (4).

Fit parameters	Intercept	Slope
$\langle N_g \rangle \sim N_h$	$-0.80 \pm 0.15$	$0.64 \pm 0.02$
$\langle N_g \rangle \sim Q$	$9.26 \pm 0.36$	$-1.03 \pm 0.05$
$\langle N_g \rangle \sim n_s$	$-2.55 \pm 0.06$	$0.78 \pm 0.02$

### 3.3. Multiplicity correlation

To acquire a deeper understanding of the mechanism of nucleus–nucleus interactions, it is necessary to investigate the correlation between the multiplicities of the different types of emitted particles and the number of projectile spectator charge,  $Q$ . For this purpose we study here the correlations between the average number of emitted grey particles released from the 3.7A GeV  $^{16}\text{O}$ -Em interactions and the target size ( $N_h$  – values), the  $Q$ -values and the number of created relativistic particles  $n_s$ .

Figs. 4a and 4b show the correlation between the  $\langle N_g \rangle$  on  $N_h$  as well as the  $\langle N_g \rangle$  dependence on  $Q$ -value are displayed.

In Fig. 4c, we also display the correlation between the  $\langle N_g \rangle$  and the number of shower particles creation, which is an important relation due to their natural dependence on the number of nucleons sharing in the hadronic matter in each event. The average values of grey particle multiplicity are strongly linearly correlated with  $N_h$ ,  $Q$ , and  $n_s$ . Table (6) shows the linear fit parameters in each case. The following observations have been obtained:

- The  $\langle N_g \rangle$  increases linearly with increasing  $N_h$  (target size) as well as  $n_s$  overall range.
- As the value of  $Q$  (number of the stripped projectile nucleons) increases, the average number of grey particle multiplicity decreases.
- The dependent of  $\langle N_g \rangle$  on the  $Q$  values is a linearly correlated for the peripheral collisions, i.e.,  $Q \geq 3$ .
- As seen from Fig. 4, the  $N_h$  and  $n_s$  values are suitable measuring experimental parameters for the impact parameter of between the projectile – target collisions.

## 4. Conclusions

We draw the following conclusions from the analysis of the experimental data on the grey particle multiplicity in the inelastic reactions of 3.7A GeV  $^{16}\text{O}$  with emulsion nuclei:

- 1) The grey particle multiplicity characteristics, as represented by the probabilities of events and average values, are the same for all studied projectiles (12C, 16O, 22Ne and 28Si), supporting the limiting fragmentation hypothesis [31].
- 2) The MD of  $g$ -particles  $P(N_g)$  in the collisions of 16O-Em and 12C-Em at 3.7A GeV has the same decay shape, which can be expressed respectively as  $P(N_g)=34.15\exp(-N_g/4.09)$  and  $35.08\exp(-N_g/4.40)$ .
- 3) The MDs of  $g$ -particles for different target sizes ( $N_h$ -value) are fitted with a Gaussian distribution, and these distributions become symmetric and of platykurtic shape. Furthermore, as the target size increases, the values of Skewness ( $S$ ) and Kurtosis ( $K$ ) decrease. Also, the scaled variance is nearly constant and less than one.

- 4) The multiplicity distribution  $P(N_g)$  of the emitted g-particles fit well with the Gauss distribution function, indicating symmetric and platykurtic behavior. Both values of Skewness (S) and Kurtosis (K) are less than zero.
- 5) The values of the average g-particle multiplicities and dispersions fluctuate through the centrality range (Q-values). The MDs for group of events have  $Q > 0$  are fitted with two Gaussian distributions (of bimodal shape) with two individual peaks. It is suggested that the grey particles may be produced from multisource system.
- 6) According to  $\langle N_g \rangle = (0.64 \pm 0.02)N_h - (0.83 \pm 0.15)$  and  $\langle N_g \rangle = (0.79 \pm 0.016)n_s - (2.55 \pm 0.06)$ , the average multiplicity values of the emitted grey particle  $\langle N_g \rangle$  increase linearly with  $N_h$  and  $n_s$ .
- 7) For all, the selectivity of the  $N_h$  particles multiplicity as an impact parameter indicator for nuclear collisions is more successful than the Q-values (charged projectile spectator nucleons).

### Declaration of Competing Interest

None.

### Acknowledgments

The authors would like to thank the director and the staff of the High Energy Laboratory at JINR for supplying the irradiated emulsion plates, as well as Prof. Pavel Zarubin. The authors would also like to thank prof. A. Abdelsalam, prof. Dr. Badawy M. Badawy and the colleagues in the emulsion group of the M.EL-Nadi High Energy Laboratory, Faculty of Science, Cairo University, for their assistance. The authors are grateful to prof. Khaled Hegab Faculty of Science, Cairo University for his language revision.

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